CBSE Board
Class XI Mathematics
Sample Paper – 6

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Find the derivative of $\cos[\sin \sqrt{x}]$.

2. Write negation of : "Either he is bald or he is tall."

3. Express $\frac{6}{-i}$ in the form of b or bi where b is a real number.

OR

Find modulus of $\sin \theta - i \cos \theta$.

4. Two coins tossed simultaneously, find the probability that getting two heads.

SECTION – B

5. A and B are sub-sets of U where U is universal set containing 700 elements. n(A) = 200, n(B) = 300 and n (A ∩ B) = 100. Find n(A' ∩ B').

6. If the function $f : N \to N$ is defined by $f(x) = \sqrt{x}$ then find $\frac{f(25)}{f(16)+f(1)}$.

OR

If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$ find $f \circ g(x)$ =?

7. Find the range of the function $f(x) = |x - 3|$

OR

Let a and B be two sets such that : $n(A) = 50$, $n(A \cup B) = 60$ and $n(A \cap B) = 10$. Find $n(B)$ and $n(A - B)$.

8. Let $A = \{6, 8\}$ and $B = \{3, 5\}$. Write $A \times B$ and $A \times A$.

9. Prove that: $\left(\frac{\cos A}{1-\tan A}\right)-\left(\frac{\sin A}{1-\cot A}\right)=\frac{1}{\cos A-\sin A}$

OR

Prove that $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$

10. By giving an example, show that the following statement is false.
"If n is an odd integer, then n is prime."

11. If a, b, c are in GP then prove that $\log a^n$, $\log b^n$ and $\log c^n$ are in AP.

12. The focal distance of a point on the parabola $y^2 = 12x$ is 4. Find the abscissa of this point.

SECTION – C

13. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ prove that $\cos\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$.

14. Let a relation R_1 on the set of R of all real numbers be defined as $(a, b) \in R_1 \Leftrightarrow 1 + ab > 0$ for all $a, b \in R$. Show that $(a, a) \in R_1$ for all $a \in R$ and $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$ for all $a, b \in R$.

15. Prove that $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}=\frac{1}{2^4}$.

16. If $x = a + b$, $y = a\alpha+b\beta$, $z = a\beta+b\alpha$ where α,β are complex cube root of unity. Show that $xyz = a^3 + b^3$.

17. A bag contains 7 white, 5 black and 4 red balls. If two balls are drawn at random from the bag, find the probability that they are not of the same colour.

18. The sums of first p, q, r terms of an AP are a, b, c respectively. Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

19. How many different numbers can be formed with the digits 1, 3, 5, 7, 9 when takes all at a time, and what is their sum?

OR

A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. Find the number of choices available to him.

20. Find all the points on the line x + y = 4 that lie at a unit distance from the line 4x + 3y = 10.

OR

Find the equation of the internal bisector of angle BAC of the triangle ABC whose vertices A, B, C are (5, 2), (2, 3) and (6, 5) respectively.

21. If in a ΔABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$ prove that a^2, b^2, c^2 are in AP.

OR

Prove that $\frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} = -\cos 2x - \cos x$

22. Find the value of k, if $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$

23. Using binomial theorem, prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25.

SECTION – D

24. Prove that

1. tan3A tan2AtanA = tan3A – tan2A – tanA
2. cotAcot2A – cot2Acot3A – cot3AcotA = 1

OR

If $A = \cos^2 \theta + \sin^4 \theta$ prove that $\frac{3}{4} \leq A \leq 1$ for all values of θ.

25. Find the mean deviation about the median for the following data:

x	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	9

26. If $\frac{\pi}{2} \le x \le \pi$ and $\tan x = -\frac{4}{3}$, find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$.

27. Solve the following system of inequalities graphically:

$3x + 2y \le 150$; $x + 4y \ge 80$; $x \le 15$; $x \ge 0$; $y \ge 0$

OR

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

28. If the coefficients of a^{r-1}, a^r and a^{r+1} in the binomial expansion of $(1 + a)^n$ are in AP, prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$

29. Along a road lie an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.

OR

Find the sum of an infinitely decreasing GP, whose first term is equal to b +2 and the common ratio to 2/c, where b is the least value of the product of the roots of the equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ and c is the greatest value of the sum of its roots.

CBSE Board
Class XI Mathematics
Sample Paper – 6 Solution

SECTION – A

1. cos[sin √ x]' = -sin (sin √ x) × cos √ x × 1/2√ x

2. He is not bald and he is not tall.

3.

$$\frac{6}{-i}=\frac{6}{-i}\times\frac{i}{i}=\frac{6i}{-i^2}=6i \qquad \because i^2=-1$$

OR

z = sin Θ – i cos Θ

Comparing with a + bi we get a = sin Θ and b = - cos Θ

$$|z|=\sqrt{a^2+b^2}=\sqrt{\sin^2\theta+\cos^2\theta}=1$$

4. Sample space S = {HH, HT, TH, TT} i.e. total number of cases = 4

Favourable cases for two heads are {HH}.

Required probability $=\frac{1}{4}$

SECTION – B

5. n(A) = 200, n(B) = 300 and n(A ∩ B) = 100

n(A' ∩ B') = n(A ∪ B)'

= 700 – n(A ∪ B)

= 700 – [n(A) + n(B) – n(A ∩ B)]

= 700 – (200 + 300 – 100)

= 700 – 400

= 300

6. f(x) = √x

f(25) = 5, f(16) = 4 and f(1) = 1

Hence,

$$\frac{f(25)}{f(16)+f(1)}=\frac{5}{4+1}=1$$

OR

$f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$
$f \circ g(x) = f[g(x)] = f(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1$

7. $f(x) = |x - 3|$
$f(x)$ is defined for all $x \in R$. Therefore, domain $f = R$
$|x - 3| > 0$ for all $x \in R$.
$0 \le |x-3| < \infty$ for all $x \in R$.
$f(x) \in [0,\infty)$ for all $x \in R$.
Range of f is $[0,\infty)$

OR

$n(A) = 50$, $n(A \cup B) = 60$ and $n(A \cap B) = 10$
$n(A \cup B) = n(A) + n(B) - n(A\cap B)$
$60 = 50 + n(B) - 10$
$n(B) = 60 - 50 + 10 = 20$
$n(A - B) = n(A) - n(A\cap B)$
$= 50 - 10 = 40$

8. $A \times B = \{(6, 3), (6, 5), (8, 3), (8, 5)\}$
$A \times A = \{(6, 6), (6, 8), (8, 6), (8, 8)\}$

9. $\text{LHS} = \left(\frac{\cos A}{1-\tan A}\right) - \left(\frac{\sin A}{1-\cot A}\right) = \left(\frac{\cos A}{1-\frac{\sin A}{\cos A}}\right) - \left(\frac{\sin A}{1-\frac{\cos A}{\sin A}}\right)$

$= \left(\frac{\cos^2 A}{\cos A - \sin A}\right) - \left(\frac{\sin^2 A}{\sin A - \cos A}\right)$

$= \left(\frac{\cos^2 A}{\cos A - \sin A}\right) - \left(\frac{\sin^2 A}{-(\cos A - \sin A)}\right)$

$= \frac{\cos^2 A + \sin^2 A}{\cos A - \sin A}$

$= \frac{1}{\cos A - \sin A}$

= RHS

$\left(\frac{\cos A}{1-\tan A}\right) - \left(\frac{\sin A}{1-\cot A}\right) = \frac{1}{\cos A - \sin A}$

OR

$\tan^4 \Theta + \tan^2 \Theta = (\tan^2 \Theta)^2 + \tan^2 \Theta$

$= (\sec^2 \Theta - 1)^2 + \sec^2 \Theta - 1$

$= \sec^4 \Theta - 2\sec^2 \Theta + 1 + \sec^2 \Theta - 1$

$= \sec^4 \Theta - \sec^2 \Theta$

10. We observe that 9 is an odd number which is not prime. Similarly, 21, 25 etc are odd integers which are not prime.

11. a, b, c are in AP

$b^2 = ac$

$(b^2)^n = (ac)^n$

$b^{2n} = a^n c^n$

$\log b^{2n} = \log a^n c^n$

$\log (b^n)^2 = \log a^n + \log c^n$

$2\log b^n = \log a^n + \log c^n$

$\log a^n$, $\log b^n$ and $\log c^n$ are in AP

12. $y^2 = 12x$ comparing with $y^2 = 4ax$, we get $a = 3$

The focal distance of any point (x, y) on $y^2 = 4ax$ is $x + a$.

The focal distance is $x + 3$.

$\therefore x + 3 = 4$

$\therefore x = 1$

Hence, the abscissa of the given point is 1.

SECTION – C

13. $\tan (\pi\cos \Theta) = \cot (\pi\sin \Theta)$

$$\frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)} = \frac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)}$$

$\sin(\pi\cos \Theta) \sin(\pi\sin \Theta) = \cos (\pi\sin \Theta) \cos (\pi\cos \Theta)$

$\cos (\pi\sin \Theta) \cos (\pi\cos \Theta) - \sin(\pi\cos \Theta) \sin(\pi\sin \Theta) = 0$

$\cos (\pi\cos \Theta + \pi\sin \Theta) = 0$

$\pi\cos \Theta + \pi\sin \Theta = \pm\frac{\pi}{2}$

$\cos \Theta + \sin \Theta = \pm\frac{1}{2}$

$$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \pm\frac{1}{2\sqrt{2}}$$

$$\cos\theta\cos\frac{\pi}{4}+\sin\theta\sin\frac{\pi}{4}=\pm\frac{1}{2\sqrt{2}}$$

$$\cos\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$$

14. For any $a \in R$ we have $1 + a^2 > 0$ for $(a, a) \in R_1$

$(a, a) \in R_1$ for all $a \in R$

Let $(a, b) \in R_1$ then,

$1 + ab > 0$

$\Rightarrow 1 + ba > 0$

$\Rightarrow (b, a) \in R_1$

$(a, b) \in R_1 \Rightarrow (b, a) \in R_1$ for all $a, b \in R$

15. Let $\alpha=\frac{\pi}{9}$

Let C = $\cos\alpha\cos2\alpha\cos3\alpha\cos4\alpha$

S = $\sin\alpha\sin2\alpha\sin3\alpha\sin4\alpha$

C × S = $(\sin\alpha\cos\alpha)(\sin2\alpha\cos2\alpha)(\sin3\alpha\cos3\alpha)(\sin4\alpha\cos4\alpha)$

$$=\frac{1}{2}\sin2\alpha\frac{1}{2}\sin4\alpha\frac{1}{2}\sin6\alpha\frac{1}{2}\sin8\alpha$$

But $\alpha=\frac{\pi}{9}$ $\therefore 9\alpha=\pi$

$\therefore \sin8\alpha=\sin(\pi-\alpha)=\sin\alpha$ and $\sin6\alpha=\sin(\pi-3\alpha)=\sin3\alpha$

C × S = $\frac{1}{2^4}\times\sin2\alpha\sin4\alpha\sin3\alpha\sin\alpha=\frac{1}{2^4}\times S$

C = $\frac{1}{2^4}$ $\quad\because S \neq 0$

16. Let $\alpha=\omega,\beta=\omega^2$ then $\beta=\alpha^2,\alpha^3=1,\alpha+\alpha^2=-1$

$$xyz=(a+b)(a\alpha+b\beta)(a\beta+b\alpha)$$

$$=(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)$$

$$=(a+b)(a^2\omega^3+ab\omega^2+ab\omega^4+b^2\omega^3)$$

$$=(a+b)(a^2+ab\omega^2+ab\omega+b^2) \quad \because \omega^3=1,\omega^4=\omega^3\times\omega=\omega$$

$$=(a+b)\left[a^2+ab(\omega^2+\omega)+b^2\right]$$

$$=(a+b)\left[a^2-ab+b^2\right]$$

$$xyz=a^3+b^3$$

17. Let A denote the event : both the balls are of the same colour.

B : both are white

C : both are red

A = B ∪ C ∪ D

P(A) = P (B ∪ C ∪ D)

= P(B) + P(C) + P(D)   B, C and D are mutually exclusive.

$$P(B) = \frac{n(B)}{n(S)} = \frac{^{7}C_2}{^{16}C_2} = \frac{21}{120}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{^{5}C_2}{^{16}C_2} = \frac{10}{120}$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{^{4}C_2}{^{16}C_2} = \frac{6}{120}$$

$$P(A) = \frac{21}{120} + \frac{10}{120} + \frac{6}{120} = \frac{37}{120}$$

Required probability = 1 – P(A) = $\frac{83}{120}$

18. Let A be the first term and D the common difference. It is given that S_p = a

$$\frac{p}{2}\left[2A+(p-1)D\right]=a$$

$$A+\frac{(p-1)}{2}D=\frac{a}{p} \quad \text{..........(i)}$$

Similarly,

$$A+\frac{(q-1)}{2}D=\frac{b}{q} \quad \text{..........(ii)}$$

and $A+\frac{(r-1)}{2}D=\frac{c}{r}$ ....(iii)

Multiplying by (i) by (q – r), (ii) by (r – p) and (iii) by (p – q) and adding

$$A(q-r+r-p+p-q)+\frac{D}{2}\left[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\right]$$

$$A(0)+\frac{D}{2}\times 0=\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

19. The total numbers 5! = 120. Suppose we have 9 in the unit's place. We will have 4! = 24 such numbers. The numbers in which we have 1, 3, 5 or 7 in the unit's place is also 4! = 24 in each case.

Hence, the sum of the digits in the unit's place in all the 120 numbers.

= 24(1 + 3 + 5 + 7 + 9) = 600

The number of numbers when we have any one of the given digit in ten's place is also 4! = 24 in each case. Hence, the sum of the digits in the ten's place = 24(1 + 3 + 5 + 7 + 9) = 600 tens = 600 × 10

Proceeding similarly, the required sum

= 600 units + 600 tens + 600 hundreds + 600 thousands + 600 ten thousands

= 600(1 + 10 + 100 + 1000 + 10000)

= 600 × 11111 = 6666600

OR

Two cases are possible:

(i) Selecting four out of first five questions and 6 out of remaining 8 questions

Number of choices = $^5C_4\,^8C_6 = ^5C_1\,^8C_2 = \dfrac{5\times8\times7}{2} = 140$

(ii) Selecting 5 out of first five questions and 5 out of reaming 8 questions.

Number of choices = $^5C_5\,^8C_5 = ^8C_3 = \dfrac{8\times7\times6}{3\times2} = 56$

Total number of choices = 140 + 56 = 196

20. Let the required point on the line x + y = 4 be (h, k) then h + k = 4.........(i)

Its perpendicular distance from the line 4x + 3y = 10 is $\dfrac{|4h+3k-10|}{\sqrt{4^2+3^2}} = 1$

|4h + 3k – 10| = 5

4h + 3k – 10 = ± 5

4h + 3k – 10 = 5 and 4h + 3k – 10 = - 5

4h – 3k = 15......(ii)

and

4h + 3k = 5........(iii)

Solving (i) and (ii) equations we get h = 3 and k = 1

Solving (i) and (iii) equations we get h = -7, k = 11

Required points are (3, 1) and (-7, 11).

OR

AB = $\sqrt{(5-2)^2+(2-3)^2}=\sqrt{10}$

AC = $\sqrt{(5-6)^2+(2-5)^2}=\sqrt{10}$

AB: AC = 1 : 1

The internal bisector AD of ∠ BAC divides BC in the ratio AB : AC = 1 : 1

So, coordinates of D are $\left(\frac{2+6}{2},\frac{3+5}{2}\right)$ = (4, 4)

Equation of AD is y – 2 = $\frac{4-2}{4-5}$(x – 5)

2x + y – 12 = 0

21. $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$

sinA = ak, sinB = bk and sinC = ck

$$\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$$

$$\frac{\sin(B+C)}{\sin(A+B)}=\frac{\sin(A-B)}{\sin(B-C)} \qquad \sin A=\sin(B+C),\sin C=\sin(A+B)$$

Sin(B + C) sin(B – C) = sin(A + B)sin(A – B)

$\text{Sin}^2B - \sin^2C = \sin^2A - \sin^2B$

$k^2b^2 - k^2c^2 = k^2a^2 - k^2b^2$

$b^2 - c^2 = a^2 - b^2$

$2b^2 = a^2 + c^2$

a^2, b^2, c^2 are in AP.

OR

$$\frac{\cos 5x+\cos 4x}{1-2\cos 3x}=\frac{\sin 3x(\cos 5x+\cos 4x)}{\sin 3x(1-2\cos 3x)}$$

$$=\frac{2\sin\frac{3x}{2}\cos\frac{3x}{2}2\cos\frac{9x}{2}\cos\frac{x}{2}}{\sin 3x-2\sin 3x\cos 3x}$$

$$=\frac{4\sin\frac{3x}{2}\cos\frac{3x}{2}2\cos\frac{9x}{2}\cos\frac{x}{2}}{\sin 3x-\sin 6x}$$

$$=\frac{4\sin\frac{3x}{2}\cos\frac{3x}{2}2\cos\frac{9x}{2}\cos\frac{x}{2}}{2\sin\frac{3x-6x}{2}\cos\frac{3x+6x}{2}}$$

$$=\frac{4\sin\frac{3x}{2}\cos\frac{3x}{2}2\cos\frac{9x}{2}\cos\frac{x}{2}}{2\sin\frac{-3x}{2}\cos\frac{9x}{2}}$$

$$=\frac{4\sin\frac{3x}{2}\cos\frac{3x}{2}2\cos\frac{9x}{2}\cos\frac{x}{2}}{-2\sin\frac{3x}{2}\cos\frac{9x}{2}}$$

$$=-2\cos\frac{3x}{2}\cos\frac{x}{2}$$

$$=-(\cos 2x-\cos x)$$ $\because \cos C - \cos D = 2\cos(C + D)/2 \cos(C - D)/2$

22. $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$

$\therefore \lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}$

$\therefore 4\times 1^3=\frac{3k^2}{2k}.$ $\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

$\therefore 4=\frac{3k}{2}$

$\therefore k=\frac{8}{3}$

23. $6^n - 5n = (1 + 5)^n - 5n$

$6^n - 5n = {}^nC_0+{}^nC_1\times 5+{}^nC_2\times 5^2+{}^nC_3\times 5^3+....{}^nC_n\times 5^n-5n$

$6^n - 5n = 1+5n+{}^nC_2\times 5^2+{}^nC_3\times 5^3+....{}^nC_n\times 5^n-5n$

$6^n - 5n - 1 = {}^nC_2\times 5^2+{}^nC_3\times 5^3+....{}^nC_n\times 5^n$

$6^n - 5n - 1 = 5^2\left({}^nC_2+{}^nC_3\times 5+{}^nC_4\times 5^2....{}^nC_n\times 5^{n-2}\right)$

$6^n - 5n - 1 = 25 \times$ an integer

$6^n - 5n = 25 \times$ an integer + 1

Hence, $6^n - 5n$ leaves the remainder 1 when divided by 25.

SECTION – D

24. 1. 3A = 2A + A

tan3A = tan(2A + A)

$$\tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

tan3A(1 – tan2AtanA) = tan2A + tanA

tan3A – tanAtan2Atan3A = tan2A + tanA

tan3A – tan2A – tanA = tanAtan2Atan3A

2.

$$\frac{\tan 3A - \tan 2A - \tan A}{\tan 3A \tan 2A \tan A} = 1$$

$$\frac{1}{\tan 2A \tan A} - \frac{1}{\tan 3A \tan A} - \frac{1}{\tan 3A \tan 2A} = 1$$

cot2AcotA – cot3AcotA – cot3Acot2A = 1

OR

$A = \cos^2\theta + \sin^4\theta = \cos^2\theta + (\sin^2\theta)^2$

$-1 \le \sin\theta \le 1$ for all θ

$0 \le \sin^2\theta \le 1$ for all θ

$(\sin^2\theta)^2 \le \sin^2\theta$ for $0 < x < 1, x^n < x$ for all $n \in N - \{1\}$

$\cos^2\theta + (\sin^2\theta)^2 \le \cos^2\theta + \sin^2\theta$ for all θ

$\cos^2\theta + (\sin^2\theta)^2 \le \cos^2\theta + \sin^2\theta$ for all θ

$A \le 1$ for all θ

$A = \cos^2\theta + \sin^4\theta$

$A = 1 - \sin^2\theta + (\sin^2\theta)^2$

$$A = 1 - \frac{1}{4} + \left(\frac{1}{4} - \sin^2\theta + (\sin^2\theta)^2\right)$$

$$A = \frac{3}{4} + \left(\frac{1}{2} - \sin^2\theta\right)^2$$

$$\left(\frac{1}{2} - \sin^2\theta\right)^2 \ge 0 \text{ for all } \theta$$

$$\frac{3}{4} + \left(\frac{1}{2} - \sin^2\theta\right)^2 \ge \frac{3}{4} \text{ for all } \theta$$

$$A \ge \frac{3}{4} \text{ for all } \theta$$

$$\frac{3}{4} \le A \le \text{ for all } \theta$$

25.

Marks	Frequency (f_i)	Cumulative frequency (cf)	$\lvert d_i\rvert=\lvert x_i - 30\rvert$	$f_i d_i$
10	7	7	20	140
15	3	10	15	45
20	8	18	10	80
25	5	23	5	25
30	6	29	0	0
35	8	37	5	40
40	4	41	10	40
45	9	50	15	135
	N = 50			$\sum f_i d_i = 505$

N = 50 ∴ N/2 = 25

The cumulative frequency just greater than N/2 is 29 and the corresponding value of x. is 30. Median = 30

$\sum f_i d_i = 505$ and N = 50

Mean deviation = $\frac{1}{N}\sum f_i\lvert d_i\rvert = \frac{505}{50} = 10.1$

26. $\tan x = -\frac{4}{3}\ ;\ \frac{\pi}{2} \le x \le \pi$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \qquad \left(\because\ \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow -\frac{4}{3} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \Rightarrow 4\left(1-\tan^2\frac{x}{2}\right) = -6\tan\frac{x}{2}$$

$$\Rightarrow 4\tan^2\frac{x}{2} - 6\tan\frac{x}{2} - 4 = 0$$

$$\Rightarrow 2\tan^2\frac{x}{2} - 3\tan\frac{x}{2} - 2 = 0$$

The equation is quadratic in $\tan\frac{x}{2}$

$$\Rightarrow \tan\frac{x}{2} = \frac{-(-3)\pm\sqrt{9+16}}{2.2} = \frac{3\pm 5}{4} = 2, -\frac{1}{2}$$

Given $\frac{\pi}{2} \le x \le \pi \Rightarrow \frac{\pi}{4} \le \frac{x}{2} \le \frac{\pi}{2} \Rightarrow \frac{x}{2} \in II^{nd}$ quadarnt

In I^{st} quadrant, $\tan\frac{x}{2} \ge 0 \Rightarrow \tan\frac{x}{2} = 2$

We know, $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow\ 1+\tan^2\frac{x}{2} = \sec^2\frac{x}{2} \Rightarrow 1+(2)^2 = \sec^2\frac{x}{2}$

$\Rightarrow \sec^2\frac{x}{2} = 5 \Rightarrow \sec\frac{x}{2} = \pm\sqrt{5} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{5}}$

In I^{st} quadrant, $\cos\frac{x}{2} \ge 0 \Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

We know $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin\frac{x}{2} = \pm\sqrt{1-\cos^2\frac{x}{2}} = \pm\sqrt{1-\frac{1}{5}} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2}{\sqrt{5}}$

In II^{nd} quadrant, $\sin\frac{x}{2} \ge 0 \Rightarrow \sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

$\therefore$ (i) $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$ (ii) $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ (iii) $\tan\frac{x}{2} = 2$

27.

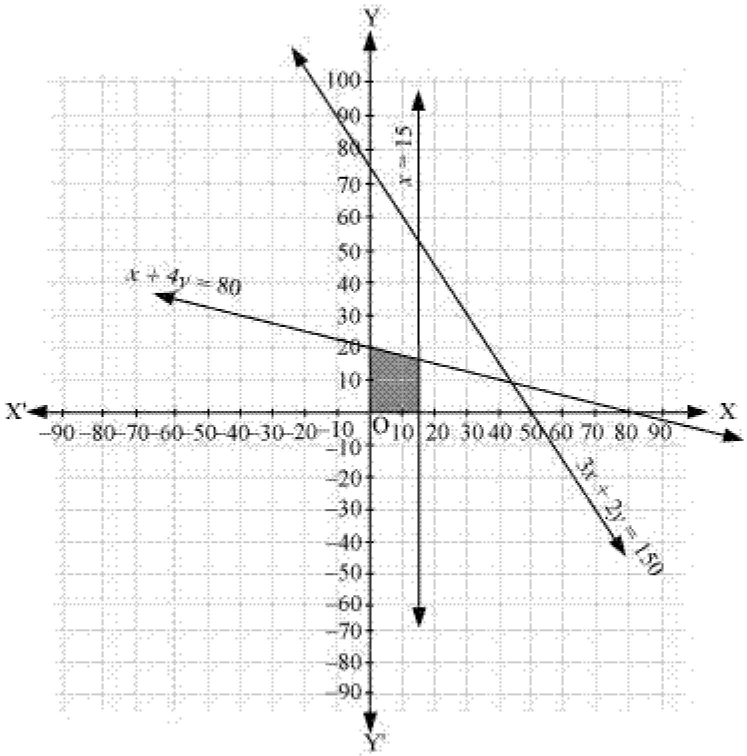
3x + 2y = 150

x	0	50
y	75	0

x + 4y = 80

x	0	80
y	20	0

x = 15



Since, $x \geq 0, y \geq 0$ every point in the common shaded region in the first quadrant including the points on the respective lines and the axes represents the solution of the given system of linear inequalities.

OR

Let x litres of water is required to be added.

Then, total mixture = x + 1125

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres.

This resulting mixture will contain more than 25% but less than 30% acid content.

30% of (1125 + x) > 45% of 1125

25% of (1125 + x) < 45% of 1125

30% of (1125 + x) > 45% of 1125

$$\frac{30}{100}(1125+x) > \frac{45}{100} \times 1125$$

(1125 + x) > 45 × 1125

30 × 1125 + 30x > 45 × 1125

30x > 45 × 1125 - 30 × 1125

30x > (45 – 30) × 1125

$$x > \frac{15 \times 1125}{30} = 562.5$$

25% of (1125 + x) < 45% of 1125

$$\frac{25}{100}(1125+x) < \frac{45}{100} \times 1125$$

$25(1125 + x) > 45 \times 1125$

$25 \times 1125 + 25x > 45 \times 1125$

$25x > (45 - 25) \times 1125$

$$x > \frac{20 \times 1125}{25} = 900$$

$562.5 < x < 900$

Thus, the required number of litres of water that is to be added will have to be more than 562.5 but less than 900.

28. The coefficients of a^{r-1}, a^r and a^{r+1} in the binomial expansion of $(1 + a)^n$ are ${}^nC_{r-1}, {}^nC_r$ and ${}^nC_{r+1}$ respectively.

It is given that ${}^nC_{r-1}, {}^nC_r$ and ${}^nC_{r+1}$ are in AP.

$$2\,{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$$

$$2 = \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r}$$

$$2 = \frac{r}{n-r+1} + \frac{n-r}{r+1}$$

$$2 = \frac{r(r+1)(n-r)(n-r+1)}{(n-r+1)(r+1)}$$

$2(n - r + 1)(r + 1) = r(r + 1) + (n - r)(n - r + 1)$

$2nr - 2r^2 + 2n + 2 = r^2 + r + n^2 - 2nr + r^2 + n - r$

$n^2 - 4nr - n + 4r^2 - 2 = 0$

$n^2 - n(4r + 1) + 4r^2 - 2 = 0$

29. Let there be $(2n + 1)$ stones. Clearly, one stone lies in the middle and n stones on each side of it in a row. Let P be the mid-stone and let A and B be the end stones on the left and right of P respectively. Clearly, there are n intervals each of length 10 m on both sides of P. Now, suppose a man starts from A. He picks up the end stone on the left of mind-stone and goes to the mid-stone, drops it and goes to $(n - 1)^{th}$ stone on left, picks it up, goes to the mid-stone and drops it. This process is repeated till he collects all stones on the left of the mid-stone at the mid-stone. So, distance covered in collecting stones on the left of the mid-stones

$= 2\{10n + 2[10(n - 1) + 10(n - 2) + ... + 10 \times 2 + 10 \times 1]\}$

Total distance covered

$= 10n + 2[10(n - 1) + 10(n - 2) + ... + 10 \times 2 + 10 \times 1] + 2\{10n + 2[10(n - 1) + 10(n - 2) + ... + 10 \times 2 + 10 \times 1]\}$

$= 4\{10n + 2[10(n - 1) + 10(n - 2) + ... + 10 \times 2 + 10 \times 1]\} - 10n$

$= 40(1 + 2 + 3 + ... + n) - 10n$

$$= 40\frac{n(n+1)}{2} - 10n$$

$= 20n^2 + 10n$

But, the total distance covered is 3 km = 3000 m

$20n^2 + 10n = 3000$

$2n^2 + n - 300 = 0$

$(n - 12)(2n + 25) = 0$

$n = 12$

Hence, the number of stones = 2n + 1 = 25

OR

$(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$

Sum of the roots = $\frac{3}{m^2+1}$ and product of the roots = $m^2 + 1$

b = least value of the product of the roots

b = least value of $m^2 + 1$

b = 1 $\quad\quad\quad\quad$ $\because m^2 + 1 > 0$ for all m

c = greatest value of the sum of the roots

c = greatest value of $\frac{3}{m^2+1}$

$\frac{3}{m^2+1}$ is the greatest when $m^2 + 1$ is the least value of $m^2 + 1$ is 1.

C = 3

So, first term of the infinite GP is b + 2 = 3 and the common ratio is 2/c = 2/3

Hence, the sum S of the infinite GP is given by

$S = \frac{3}{1-\frac{2}{3}} = 9$